Reg. No. : ....

Name : ....

Fourth Semester B.Tech. Degree Examination, June 2016
(2013 Scheme)
13.401 : PROBABILITY, RANDOM PROCESSES AND NUMERICAL TECHNIQUES (FR)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. Let X be a random variable with E(X) = 1 and E[X(X – 1)] = 4. Find V(X) and V(2 – 3X).

2. The joint pdf of the random variables X and Y is given by

$$f(x,y)=\begin{cases}kxye^{-(x^2+y^2)}, & x>0,\ y>0\\ 0, & \text{otherwise}\end{cases}$$

Find the value of k.

3. For any two random variables X and Y, show that Cov(aX, bY) = abCov(X,Y) where a and b are constants.

4. Define the power spectral density (psd) function of a stationary process. Write the Wiener-Khinchin relations.

5. Find the interpolating polynomial to find y from the following data.

x :	0	1	2	5
y :	2	3	12	147

P.T.O.

PART – B

Answer **one full** question from **each** module. **Each** question carries **20** marks.

Module – I

6. a) If the number of telephone calls coming to a telephone exchange between 9 AM and 10 AM and between 10 AM and 11 AM are independently distributed Poisson variables with parameters 2 and 6 respectively, find the probability that more than 5 calls arriving between 9 AM and 11 AM to that exchange.

   b) The duration of a telephone call in minutes follows exponential distribution with mean 4. Find the probability that a call will last (i) more than 8 minutes (ii) between 3 and 6 minutes and (iii) less than 2 minutes.

   c) In a normal population with mean 15 and standard deviation 3.5, it is found that 647 observations exceed 16.25. What is the total number of observations in the population ?

7. a) In a family of 5 children, the probability that there are 3 boys is twice as the probability that there are two boys in the family. Out of 2500 such families with 5 children each, in how many families do you expect to have (i) exactly two boys (ii) no boys and (iii) no girls.

   b) Buses arrive at a specified stop at 15 minutes interval starting from 7AM. Thus they arrive at 7, 7.15, 7.30, 7.45 and so on. If a passenger arrives at the stop at a random time that is uniformly distributed between 7 and 7.30 AM, find the probability that he waits (i) less than 5 minutes for a bus and (ii) at least 12 minutes for a bus.

   c) X and Y are two independent normally distributed random variables with means 45 and 44 and standard deviations 2 and 1.5 respectively. Find the probability that randomly chosen values of X and Y differ by 1.5 or more.

Module – II

8. a) The join pdf of the random variables X and Y is given by

$$f(x,y) = \begin{cases} 8xy, & 0 \le x \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

   Find (i) $P(X < \frac{1}{2} \text{ and } Y < \frac{1}{4})$ (ii) the marginal and conditional densities of X and Y. Are the variables independent ? Give reasons for your answer.

b) Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls. Let X be the number of white balls drawn and Y be the number of red balls drawn. Find the joint pdf of X and Y.

c) Let X be a uniformly distributed random variable over (–1, 1) and $Y = X^2$. Show that X and Y are uncorrelated. Are they independent ?

9. a) Verify whether the random process {X(t)} defined by $X(t) = Y \cos \omega t$ where Y is a uniformly distributed random variable in (0, 1) and ω is a constant is an SSS process.

b) A stationary process X = {X(t)} with mean 3 has autocorrelation function $R(\tau) = 16 + 9e^{-|\tau|}$. Find the standard deviation of the process.

c) If $X(t) = a\cos(\omega t + \theta)$ and $Y(t) = b\sin(\omega t + \theta)$ where a, b and ω are constants and θ is a random variable uniformly distributed in $(0, 2\pi)$, show that X(t) and Y(t) are jointly wide sense stationary.

Module – III

10. a) Define a Poisson Process. Is it an SSS process ? Justify your answer.

b) Define a mean ergodic process. Let Z(t) = YX(t) where X(t) is a mean ergodic process with constant mean $\mu_X \neq 0$ and Y is a random variable independent of X(t) for every t with E(Y) = 0. Is Z(t) mean ergodic ?

11. a) The autocorrelation function of a wide sense stationary process is given by $R(\tau) = e^{-\alpha|\tau|}(1 + \alpha|\tau|)$. Determine the power spectral density of the process.

b) The power spectral density of a wide sense stationary process X(t) with mean zero is given by

$$S(\omega) = \begin{cases} k & \text{if } |\omega| < \omega_0 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant. Show that X(t) and $X\left(t + \frac{\pi}{\omega_0}\right)$ are uncorrelated.

Module – IV

12. a) Find the smallest positive root of $x^2 - \log x - 12 = 0$ using Regula-falsi method correct to two decimal places.

b) Solve the following system of equations by Gauss elimination method correct to two decimal places.

$$x + y + z = 9$$
$$2x - 3y + 4z = 13$$
$$3x + 4y + 5z = 40$$

c) Find a negative root of $x^3 - 2x + 5 = 0$ by Newton-Raphson method correct to two decimal places.

13. a) Using Newtons interpolation formula find y when (i) x = 48 and (ii) x = 84.

x :	40	50	60	70	80	90
y :	184	204	226	250	276	304

b) Find $\int_0^1 \frac{x^3}{1+x^3} dx$ dividing the interval of integration into 4 equal parts by

i) Trapezoidal rule

ii) Simpson's one third rule.
